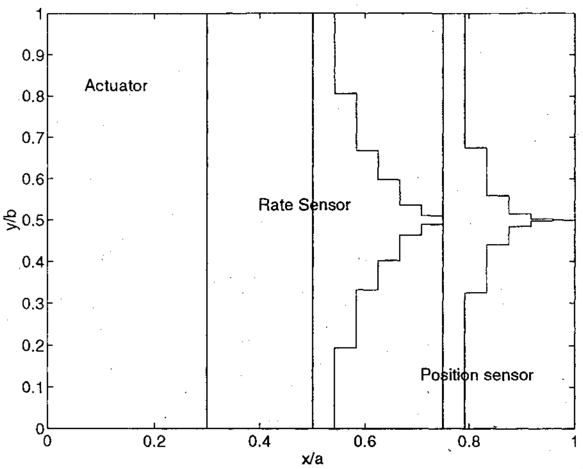

Fig. 3 Shapes and locations of the actuator and position and rate sensors.

The shape of sensor is given by C_d. If all of the elements of $\bar{C}_d$ do not have the same sign, then the location and width of the stripes need to be adjusted, until all of the elements of $\bar{C}_d$ have the same sign.

Simulation and Discussion

Numerical analysis is based on a simply supported square aluminum panel with two piezoelectric laminates that cover the two surfaces of the panel. The physical parameters and the geometry of the panel can be found in Ref. 3. The one-patched actuator is located at the leading edge with the width of 30% panel length. The key for this simulation is to find the location and the shape of the sensor to make all of the elements of vector $\bar{C}_p$ have the same sign. Then, the desired control performance can be achieved through a constant gain direct feedback controller. If the desired control is optimal control, the optimal control performance can be achieved by using the shaped sensors with a constant gain direct feedback controller.

When optimal control is used,[3] the feedback gain varies with the dynamic pressure. Then, the sensor design needs to be updated with the change of the dynamic pressure. It might be interesting to see the constant gain direct feedback control performance by using the shaped sensors designed at the certain dynamic pressure. Here, the dynamic pressure $\lambda = 1500$ is used. Since six modes are used to represent the deflection of the panel in this simulation, the number of sensor stripes should be larger than 12 to get both position and rate sensors. Here, the piezoelectric layer for sensing is divided into 24 stripes. The shape of the position sensor is designed at location $x/a = 0.75$–1. The shape of the rate sensor is designed at location $x/a = 0.5$–0.75. The calculation shows that all of the elements of $\bar{C}_p$ have the same sign with these sensor locations chosen. Then, the location and width of the sensor stripes need not to be adjusted. Figure 3 shows the shapes and locations of the actuator, position, and rate sensors. The position feedback gain k_p is −449,060, and the rate feedback gain k_d is 0.14954. The maximum suppressible dynamic pressure $\lambda_{\max}$ can reach 1900, which is larger than $\lambda_{\max} = 1744$ obtained by using optimal control.[3] The flutter free region is enlarged about four times of $\lambda_{cr} = 512$. It should be noted that the presented concept can be applied to some other area for the shaped sensor design.

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Optimal Sensor Location in Active Control of Flexible Structures

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Introduction

THIS paper presents a new algorithm for the optimal distribution of concentrated sensors to measure displacements or strains in flexible structures; this procedure is applicable in quasistatic shape control (e.g., parabolic mirrors). The proposed approach is based on the analysis of the properties of a linear matrix equation describing the modal strain decomposition, where observable as well as interfering modes are taken into account. To identify n eigenmodes, n sensors are required, and their location should be selected in such a way that the linear system providing the desired n modal amplitudes is well conditioned and the influence of upper modes (observation spillover) is minimized.

Problem Formulation

In this section the problem of static identification of n eigenmodes of deformation when $m + n$ may be relevant is addressed. It can be formulated as the following linear matrix equation:

$$A(\xi)\gamma = b - r(\xi) \tag{1}$$

where the columns of the $n \times n$ matrix A are vectors describing shape functions (at the points where the sensors are located) of the chosen modes; such positions are characterized by the coordinates ξ_i, $\xi^T = (\xi_1, \ldots, \xi_n)$. The vectors γ and b contain the n modal amplitudes to be identified and the n measured quantities, respectively. The vector r represents the perturbation by all the eigenmodes not included in A and is given by

$$r(\xi) = B(\xi)\beta + \rho \tag{2}$$

where the columns of the $n \times m$ matrix B are vectors whose components are the values (at the sensor points) of the m unwanted modes, the m-vector β contains their amplitudes, and the n-vector ρ represents the negligible influence of the residual eigenmodes (other than the $m + n$ ones).

The perturbation vector r changes the solution of the system (1). The optimization problem consists in obtaining ξ so that the relative change $\|\Delta\gamma\|_2/\|\gamma\|_2$ is minimal. Here $\|\cdot\|_2$ is the Euclidean norm ($\|x\|_2^2 := x^T x$). The ratio $\|\Delta\gamma\|_2/\|\gamma\|_2$ can be bounded[1] as follows:

$$\frac{\|\Delta\gamma\|_2}{\|\gamma\|_2} \le \mathrm{cond}[A(\xi)]\, \|r(\xi)\|_2 \frac{1}{\|b\|_2} \tag{3}$$

where $\mathrm{cond}(A) := \|A\|_2\|A^{-1}\|_2$ is the condition number of the matrix A; in this equation $\|\cdot\|_2$ is the matrix norm subordinated

Received May 30, 1995; revision received Dec. 1, 1995; accepted for publication Dec. 5, 1995.

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to the Euclidean vector norm ($\|A\|_2 := \sup \|Ax\|_2/\|x\|_2$ provided $\|x\|_2 \neq 0$). The definition of $r(\xi)$ in (2) (ignoring the influence of ρ) shows that

$$\|r(\xi)\|_2 \leq \|B(\xi)\|_2\|\beta\|_2 \tag{4}$$

Therefore, as the vectors b and β are out of control, the objective function f to be optimized can be defined by

$$f(\xi) := \text{cond}[A(\xi)]\,\|B(\xi)\|_2 = \frac{\sigma_1[A(\xi)]\sigma_1[B(\xi)]}{\sigma_n[A(\xi)]} \tag{5}$$

where $\sigma_1(\cdot)$ and $\sigma_n(\cdot)$ are the biggest and the smallest singular value of a matrix, respectively.[2] The minimization of f ensures both a well-conditioned matrix A and a weak influence of the m interfering modes (through matrix B).

If there is no sensor on the boundary of the structure (i.e., $\xi_i \neq 0$ for $i = 1, \ldots, n$) and all of them are placed in different positions (i.e., $\xi_i \neq \xi_j$ for $i \neq j$), then since the eigenmodes are orthogonal, it follows that the matrix A is nonsingular. This means that the function $f(\xi)$ is well defined.

For the working set to be compact and not have too small amplitudes (which are difficult and costly to measure), the following nonlinear constraint is taken into account:

$$\sigma_n[A(\xi)] \geq \delta > 0 \tag{6a}$$

where δ is a lower bound for σ_n. To eliminate the equivalent solutions arising from the prenumbering of the decision variables ξ_i, the following linear constraints are also taken into account:

$$0 \leq \xi_1 \leq \cdots \leq \xi_n \leq \bar{\xi} \tag{6b}$$

where $\bar{\xi}$ is related to the way in which the coordinates ξ_i have been defined; without restricting the scope of the study they can be normalized as $\bar{\xi} = 1$.

The optimization problem is formulated as the minimization of the objective function f in (5) subject to the constraints in (6).

Numerical Solution

The optimization problem is numerically solved by the augmented Lagrangian method.[1] This procedure finds local optimal points and can be applied only if both constraints and objective functions are differentiable (at least almost everywhere) on the feasible region. The differentiability of the greatest-eigenvalue function,[3] matrix inverse theory,[2] and the constraints (6) show that the conditions for applying the augmented Lagrangian method are fulfilled.

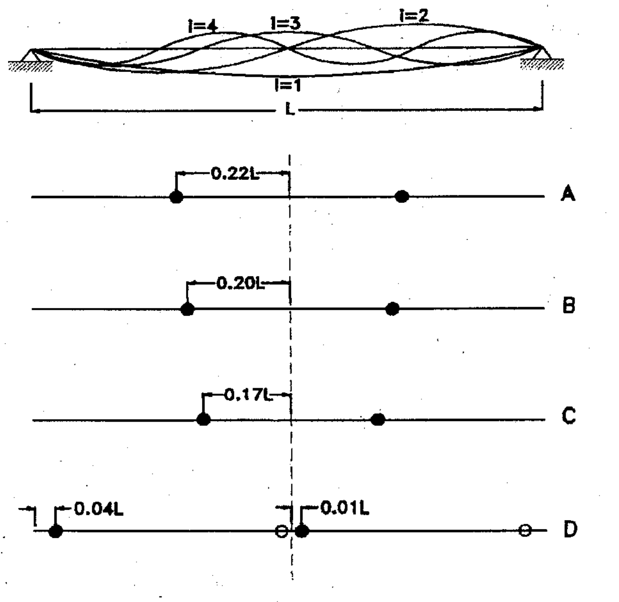

Fig. 1 Simply supported beam: eigenmodes and locations of sensors.

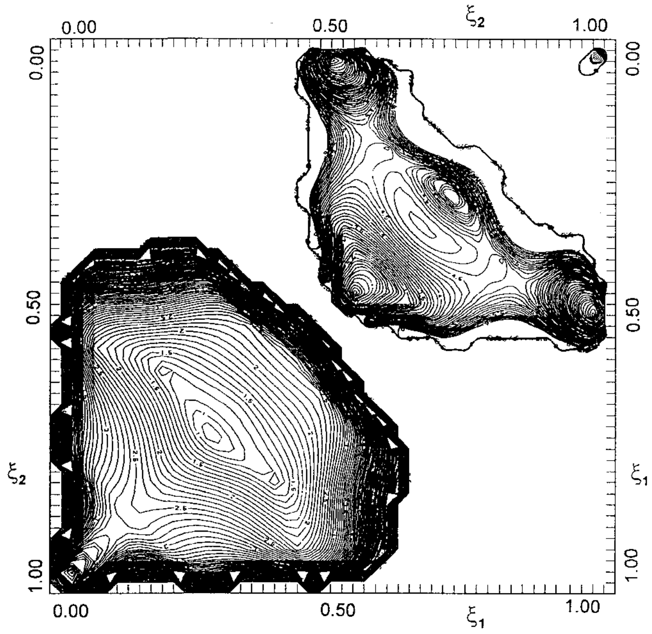

Fig. 2 Objective function, cases A and D.

As the objective function f can have local minima, additional procedures (based on interval analysis) are used to solve the global optimization problem.[4] The situating of the local optimal points in the two-dimensional problem described in the next section can be examined on the basis of Fig. 2.

All local and global optimal points are found in the interior of the feasible region, which shows that the problem of choosing δ in Eq. (6a) is not critical (provided that it is greater than zero).

Sample Problem

The optimal location of n sensors measuring local deflections in a pin-ended beam is selected (see Fig. 1). If the cross section is constant, the ith eigenmode is given by the closed-form expression $y_i(x) = a_i \sin(\pi i x/L)$, as represented in Fig. 1. In this particular case, the matrices A and B are

$$A(\xi) = \begin{pmatrix} a_1 \sin \pi\xi_1 & \cdots & a_n \sin \pi n\xi_1 \\ \vdots & & \vdots \\ a_1 \sin \pi\xi_n & \cdots & a_n \sin \pi n\xi_n \end{pmatrix}$$

$$B(\xi) = \begin{pmatrix} a_{n+1} \sin \pi(n+1)\xi_1 & \cdots & a_{m+n} \sin \pi(m+n)\xi_1 \\ \vdots & & \vdots \\ a_{n+1} \sin \pi(n+1)\xi_n & \cdots & a_{m+n} \sin \pi(m+n)\xi_n \end{pmatrix} \tag{7}$$

where $\xi_i = x_i/L$. Here x_i are the coordinates of the sensors, and L is the beam length. The previous knowledge of the proportions among the modes can be used to select the values of n, m, and amplitude weighting factors a_i.

Four cases (A, B, C, and D) are analyzed. In all of them, $n = m = 2$. The values of the weighting factors a_i as well as the selected values of normalized coordinates ξ_i and the objective function f are shown in Table 1.

The values of a_i in Table 1 show that case A could correspond to a situation where no information about the relative importance among the modes is available. In case B the influence of upper modes is lessened, and in case C the fourth mode is not considered (it is equivalent to the case $m = 1$). The second mode is strongly magnified in case D, which could correspond to the expectation of antisymmetrical deformation.

The values of the objective function f are shown in Fig. 2 for cases A and D. The surfaces of f in terms of ξ_1 and ξ_2 are represented

Table 1 Modal weighting factors (a_i), selected locations (ξ_i), and objective function (f)

Case	a_1	a_2	a_3	a_4	ξ_1	ξ_2	f
A	1	1	1	1	0.2837	0.7143	0.7652
B	1	$\frac{1}{4}$	$\frac{1}{9}$	$\frac{1}{16}$	0.2985	0.7015	0.1712
C	1	$\frac{1}{4}$	$\frac{1}{9}$	0	0.3333	0.6667	0
D	1	4	1	1	0.0402	0.5050	1.0721

through contour lines. The selected values of ξ_i correspond to the local minima. In case C the minimization of the objective function leads to the trivial solution $\sigma_1[\boldsymbol{B}(\boldsymbol{\xi})] = 0$, i.e., $f = 0$. The feasible area of the design variables ξ_1 and ξ_2 is limited by the constraint (6a). The selected locations for cases A, B, C, and D are represented in Fig. 1.

The results from Fig. 1 (or Table 1) show that the optimal locations are sensitive to the assumed proportions among a_i.

In case C the selected positions are the nodal points of the third mode (the interfering one) which corresponds to $f = 0$. In fact, the definition of f in (5) shows that, if $m = 1$, its minimum corresponds to $\sigma_1[\boldsymbol{B}(\boldsymbol{\xi})] = 0$, i.e., the sensors are placed at points with zero amplitude in the $(n + 1)$th mode. This means that the result in case C is obtained regardless of the values of a_1, a_2, and a_3 (provided that they are not 0). Nevertheless, it is not guaranteed that the matrix A is well conditioned.

In case D the sensors are not symmetrically located because of the asymmetry in the deflection. However, local minima ($\xi_1 = 0.29$, $\xi_2 = 0.71$ and $\xi_1 = 0.46$, $\xi_2 = 0.54$) corresponding to symmetric solutions exist as well. A solution close to the first one would be chosen if $a_3 = a_4 = 0$.

Conclusions

This paper presents a new criterion for selecting the positions of concentrated sensors for the measurement of the modal components of distortions in flexible structures. The optimization criteria are the minimization of the condition number of a matrix related to the modal decomposition and of the perturbation caused by upper modes.

Initial results show that the obtained locations depend on the expected proportions among the modes. The problem is sensitive to the influence of the interfering ones.

If the number of interfering modes is taken as equal to 1, the optimization process is strongly affected by the minimization of the influence of that mode, and so the selected locations are its nodal points. In that case, the condition number of the main matrix is not guaranteed to be small. This drawback may be avoided by taking into account several interfering modes.

If no certainty about the dominant mode is available, several redundant sets of sensors (corresponding to different expected proportions among the modes) could be installed. Relative weighting among its measurements might be adapted on-line according to real-time observed proportions.

Acknowledgments

This paper is supported by the Spanish Government (DGICYT Research Project PB91-0591 and Grant SAB93-0215) and NATO (Collaborative Research Grant OUTREACH CRG 950316). The numerical calculations were performed at Poznań Supercomputing and Networking Center (Grant KBN nr. 8 T11F 010 08p02).

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Simplified Calculation of Eigenvector Derivatives with Repeated Eigenvalues

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Introduction

DERIVATIVES of eigenvalues and eigenvectors with respect to structural parameters play an important role in structural design, identification, and optimization, and so research in this area has drawn the attention of many scholars for a long time. An excellent survey paper by Adelman and Haftka[1] summarizes the progress in the study of the modal sensitivities through 1986.

In 1976, Nelson[2] presented an efficient algorithm for computing derivatives of eigenvectors for the general real eigensystems with nonrepeated eigenvalues. The greatest advantage of Nelson's method is that it preserves the symmetry and bandedness of the original eigensystem and requires the knowledge of only those eigenvectors that are to be differentiated. But it cannot directly deal with cases of repeated eigenvalues that often occur in many practical engineering structures, such as the wheelsets on train, rotor systems, and geometrically symmetric structures. Ojalvo,[3] Mills-Curran,[4] Dailey,[5] and Shaw and Jayasuriya[6] extended Nelson's method for solving the derivatives of eigenvalues and eigenvectors of the structure with repeated eigenvalues. Hou and Kenny[7] introduced an alternate eigenvector derivative matrix formulation and reparameterized the multivariable eigenproblem into an eigenproblem that is in terms of a single positive-valued design parameter to obtain the eigenvalue and eigenvector approximate analysis for repeated eigenvalue problems. Recently, Liu et al.[8] considered the contribution of the truncated modes to eigenvector derivatives and proposed a more accurate method to calculate the eigenvector derivatives.

Assuming that the repeated-root modal derivatives have the form

$$\frac{\partial X}{\partial p} = X' = V + XC$$

The focus of the extended Nelson's method[3–5] is concentrated on the problem of how to determine the unique eigenvectors X and its coefficient matrix C, not on the problem of how to determine the matrix V. For solving the matrix $V = [v_1 \mid v_2 \mid \cdots \mid v_m]$, Ojalvo[3] and Dailey[4] suggested that v_i can be obtained by appropriately eliminating some rows and columns of the system matrix $(K - \lambda M)$ and solving the resultant linear equation. Unfortunately, this method may fail in some circumstances.[9] A more rigorous approach has been presented by Mills-Curran.[4] But it is difficult to implement on computer.

This paper deals with this problem and proposes an efficient method to determine the first part of the repeated-root eigenvector derivative (e.g., the matrix V) by introducing a set of nonmodal vectors that are easily obtained from the modes associated with the repeated eigenvalue. Those vectors are orthogonal to the repeated-root eigenvectors and consist of a basis of N-dimensional space together with the known repeated-root eigenvectors, and so vectors v_i can be expressed as a linear combination of those nonmodal vectors. Consequently, the coefficients are determined. As a numerical example, a cantilever beam is presented to illustrate the application of the proposed procedure. Accuracy of the new method is examined by a finite difference scheme.

Received Jan. 10, 1994; revision received May 1, 1995; accepted for publication May 13, 1995.

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